Max-n: A system for quantifying transfinite ordinal hierarchies with finite values

June 7, 2023

The **Max-n system** refers to an idea for a numerical scheme where only n-1 positive integers exist, with the first number after the highest integer being symbolized by ω. This allows for reaching ω from 0 in exactly n steps. Using this system, it becomes possible to reach any countably infinite ordinal in a finite number of steps (where a step can be thought of as adding 1 to the previous number). This system is designed for ordinal arithmetic only; It is only functional for ordinals below $\epsilon 0$ and is not defined for functions like the Veblen function or other ordinal notations.

Consider Max-10 as an example. Within this system, 9 signifies the highest positive integer, with ω following immediately after. This is a close analog to the base-10 system. Using Max-n one can assign a "value" to each ordinal, dependent on the steps required to reach a specific ordinal. For instance, within Max-10, ω would have a value of 10, and $\omega 2$, 20 (because only 10 potential values exist between ω and $\omega 2$, the greatest of which is $\omega + 9$).

In the general Max-n structure, ω^x and $\omega^{\omega x}$ are respectively equivalent to n^x and n^{nx}, establishing an interesting and helpful bijective relation. A salient rule of this system states that the value of any ordinal less than $\epsilon 0$ can be found in Max-n simply by substituting its ω values with n, rendering the computation of Max-n values relatively simple.

A side effect of this system is observable in Max-2: Since $2 * 2$, 2^2, and $2 \uparrow\uparrow 2$ all equate to 4, the only four viable steps within this system are 0, 1, ω, and $\omega + 1$. After $\omega + 1$, $\omega + \omega$ should equal $\omega 2$, however, as 2 isn't reachable in Max-2, $\omega + \omega$ equals $\omega * \omega$ which equals ω^ω, and so forth.

This system has a close relation with the slow-growing hierarchy: The value of ordinal n in Max-m is exactly equal to $g_n(m)$ in SGH.

$\epsilon 0$ can be reached in Max-n after $n \uparrow\uparrow n$ steps ($\epsilon 0$ can be informally thought of as $\omega \uparrow\uparrow \omega$). The first 5 values of $\epsilon 0$ in Max-n are 1, 4, 7.62e12, 1e8.07e153, and 1e1e1.33e2,184. Counting to $\epsilon 0$ in Max-3, assuming one step is written down every 3 seconds, would take approximately 1,900,000 years.

Values in Max-n can informally be extended beyond $\epsilon 0$ by using non-standard definitions of ordinal tetration, pentation, etc. In this case, the most likely value of $\zeta 0$ ($\phi(2, 0)$ using the Veblen function) in Max-10 would be $10 \uparrow\uparrow\uparrow 10$. Following this, the most likely value of $\phi(m, 0)$ in Max-n is n{m+1}n, where a represents a up arrows.

1 Max-10 function

Max-10 function, or $M_{10}(n)$, is a function that converts an ordinal into its corresponding value in Max-10. This function works simply for ordinals up to ϵ0, and can be extended using assumed non-standard definitions of ordinal operators beyond exponentiation. The function can be extended further using the Veblen function and a hypothetical expansion of the Max-n system that there is currently no definition for. Assuming that the value of $\phi(2, 0)$ is 10 $\uparrow\uparrow\uparrow$ 10 and that the value of $\phi(n, 0)$ is 10{n+1}10 for n¡10, the value of $\phi(\omega, 0)$ would be 10{11}10, or {10,10,11} in BEAF. Thus, $M_{10}(\phi(\omega, 0))$ = {10,10,11}. Following this, $M_{10}(\phi(\phi(\omega, 0), 0))$ = {10,10,{10,10,11}} and $M_{10}(\phi(1, 0, 0))$ = {10,10,1,2}.